Lecture 8

Thursday, May 12, 2022 2:32 PM

* Prayer

* Spiritual thought

* Geometric meaning of partial derivatives ....

Ex $f(x,y) = xy + y^2$

$\frac{\partial f}{\partial x}(1,2) = ?$, $\frac{\partial f}{\partial y}(1,2) = ?$

Another notation for $\frac{\partial f}{\partial x}$ is f_x, for $\frac{\partial f}{\partial y}$ is f_y.

Higher derivative :

$f_x, f_{xx}, f_{xy}, f_{xyx}, \ldots$ same as $\frac{\partial f}{\partial x}, \frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial x \partial y}, \frac{\partial^3 f}{\partial x \partial y \partial x}, \ldots$

Ex $f(x,y) = x^y$

Find f_{xy} and f_{yx}.

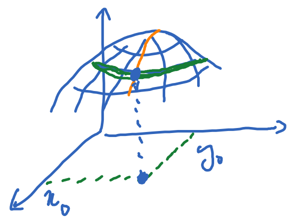
Clairaut's theorem : partial derivatives don't depend on the order of differentiation.

Why is $f_{xyxy} = f_{yxyx}$?

* Applications of partial derivatives :

- Tangent plane
- Linear approximation
- rate of change in a certain direction
- optimisation problems

* Tangent plane:



The red curve has equation:

$$\begin{cases} x = t \\ y = y_0 \\ z = f(t, y_0) \end{cases}$$

and has tangent vector at (x_0, y_0): $v = \left(1, 0, \frac{\partial f}{\partial x}(x_0, y_0)\right)$

The green curve has tangent vector at (x_0, y_0): $w = \left(0, 1 \ \frac{\partial f}{\partial y}(x_0, y_0)\right)$

The tangent plane has a normal vector $v \times w = (-f_x, -f_y, 1)$.

The tangent plane has eq.

$$(-f_x)(x - x_0) + (-f_y)(y - y_0) + (z - z_0) = 0$$

$$\rightsquigarrow \quad z = f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0).$$

Ex $\quad f(x, y) = x^2 + 2y^2$

Find tangent to this elliptic paraboloid at $(1, -1, 3)$.

Ex $\quad x^2 + 2y^2 + 2z^2 = 1$

Find tangent plane to this ellipsoid at $\left(\frac{1}{2}, -\frac{1}{2}, \frac{1}{2}\right)$.

* Linear approximation:

$$y = f(x) \rightsquigarrow \underbrace{dy}_{\text{change in } y \text{ cause by change in } x} = f'(x)\underbrace{dx}_{\text{change in } x}$$

change in y cause by change in x

change in x

$z = f(x,y)$

$$dz = f_x\,dx + f_y\,dy$$

change in z due to change in x and y

change in z due to change in x (with y fixed)

change in z due to change in y (with x fixed)

dz: total differential

$f_x\,dx$, $f_y\,dy$: partial differential

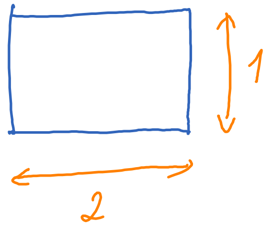
Linear approximation:

$$f(x_0+h, y_0+k) - f(x_0,y_0) \approx f_x(x_0,y_0)\,h + f_y(x_0,y_0)\,k$$

In other words,

$$f(x,y) \approx f(x_0,y_0) + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)$$

Ex



Area $= 1 \times 2 = 2$

If the sides of the rectangle are measure with error: $x = 1 + h$, $y = 2 + k$

where $-0.01 \leq h, k \leq 0.01$.

$$\text{Area} = xy = f(x,y) \approx f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(x_0, y_0)(y - y_0)$$

$$= 1(2) + 2h + 1(k)$$

$$= 2 + \underbrace{2h + k}$$

$$-0.03 \leq \cdots \leq 0.03$$